Optimal Charging of Ultracapacitors During Regenerative Braking

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***Abstract*— Finding the optimal charging profile of an ultracapacitor energy storage system during a regenerative braking event is the focus of this paper. After showing that resistive losses can be high during a high power regeneration event, we formulate the charging problem in an optimal control framework with the objective of maximizing the energy recuperated into the ultracapacitor bank while satisfying braking power demands. We employ Pontryagin's maximum principle to understand the necessary conditions the solution should satisfy and use numerical techniques to find such optimal solution(s). The result should provide more insight into the maximum achievable regeneration efficiency with ultracapacitors under different braking conditions and can also aid in sizing an ultracapacitor energy storage system and the associated power electronics device.**

I. Introduction

In fully electric or hybrid electric vehicles, the capability to recover part of vehicle's kinetic energy during braking events plays an important role in increased energy efficiency of the vehicle. The role of regeneration may be even more important when ultracapacitors are used for energy storage, because of their high power but low energy density. High resistive losses in this case can substantially reduce the recuperation efficiency. To provide a better understanding of the fundamental limitations with ultracapacitor energy storage and to confirm what the best charging patterns are, we formulate the charging problem as an optimal control problem in which the goal is to maximize the energy recuperated into the ultracapacitor bank while satisfying braking requests. Pontryagin's maximum principle provides the necessary conditions that candidate solutions should satisfy. We resort to numerical techniques to find the optimal charging current profile in a regenerative braking event.

We start by showing the (known) fact that when a discharged ultracapacitor is fully charged by a constant voltage source, exactly 50 percent of the charging energy is lost to the line resistance independently of how small this resistance may be. In *stand-alone* ultracapacitor charing, it is straightforward to show, via application of Pontryagin's Maximum Principle, that charging with constant current is energy optimal [1]. However, it is not obvious if constant charging is energy optimal during *regenerative charging* due to the coupling with vehicle velocity and motor dynamics and contraints. While the optimal control formulation of the problem is relatively straight-forward, its solution is not, as now the necessary conditions for optimality are more complex. In this paper we resort to a numerical technique to find solutions that satisfy the optimality necessary conditions and present the preliminary results.

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In our group, we have previously demonstrated via detailed full driving cycle simulation analysis that, with the power assistance provided by a stand-alone bank of ultracapacitors the fuel economy of heavy trucks could be improved up to 40% in a stop and go driving cycle [2]. For mild hybrid passenger cars we found 15% gain in fuel economy in city driving was possible [3]. However it is difficult to isolate the real efficiency bottlenecks via a full-scale powertrain simulation over an entire driving cycle. The objective of this paper is to provide more in-depth understanding of the fundamental efficiency limitations during an isolated regenerative braking event.

As a recent article [4] correctly points out there is much ”confusion” and ”uncertainty” in the literature regarding power, cost, and weight advantages that can be gained with ultracapacitors over advanced batteries. Perhaps because of these uncertainties, ultracapacitors have not yet made their way to mass produced hybrid vehicles. Only at a limited scale they have been used as the main energy storage units in fleets of buses [5]. Quantifying how efficiently ultracapacitors can store (and release) energy provides a better understanding of their potentials and can also aid in sizing an ultracapacitor energy storage system.

II. Motivation

It is well known that charging a capacitor and similarly an ultracapacitor, from zero charge to full charge, with a constant voltage source results in 50% energy loss irrespective of the internal and line resistances. This can be easily shown by writing the differential equation governing the ultracapacitor stored charge, $q(t)$, for the circuit shown in Figure (1):

$$RC\frac{dq}{dt} + q = CE_{dc} \tag{1}$$

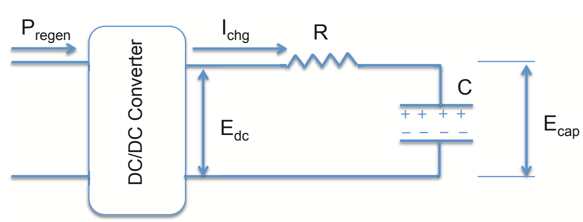

Fig. 1. Schematic of the charging circuit.

where C is the capacitance of the ultracapacitor, R is the lumped resistance of the ultracapacitor and connecting cables, and E_{dc} is the charging voltage. If E_{dc} remains constant over time, the solution to the above differential equation from a zero initial charge condition can be obtained to be:

$$q(t) = C.E_{dc}.\left(1 - e^{-t/RC}\right) \quad (2)$$

and the charging current i_{chg} is then:

$$i_{chg}(t) = \frac{E_{dc}}{R} e^{-t/RC} \quad (3)$$

One can find the resistive energy loss by integrating the resistive power loss Ri_{chg}^2 over the entire charging interval $[0, +\infty)$,

$$\frac{E_{dc}^2}{R} \int_0^\infty e^{-2t/RC} dt$$

to be equal to $\frac{1}{2}CE_{dc}^2$ which is also equal to the total energy stored in the ultracapacitor. In other words the efficiency of charging an empty ultracapacitor with a constant voltage source is 50% independently of resistance R. Note that the charging efficiency depends on both the initial and final state of charge; for example charging an ultracapacitor from half to full charge with constant voltage has an efficiency of 75 percent[1].

It is then natural to ask what charging current (or voltage) profile maximizes the charging efficiency. That is the current that would charge the ultracapacitor to a desired level of charge with minimum resistive losses. Let's choose the optimization variable to be the charging current, $u_1(t) = i_{chg}(t)$. The ultracapacitor state of charge $SOC = \frac{q(t)}{q_{max}}$ quantifies the amount of charge stored in the ultracapacitor bank normalized by the maximum charge it can accept q_{max}. We can think of ultracapacitor's state of charge as a dynamic state x_1 with the following dynamics:

$$\frac{d}{dt}x_1 = \frac{u_1}{CE_{max}} \quad (4)$$

where E_{max} is the voltage across the ultracapacitor at maximum charge. Let's assume the ultracapacitor is initially free of charge $x_1(0) = 0$ and in t_f units of time is charged to its final desired state of charge SOC_f, therefore $x_1(t_f) = SOC_f$. The optimal input $u_1(t)$ is one that minimizes the resistive losses in the time period $[0, t_f]$ characterized by the following cost function:

$$J = \int_0^{t_f} Ru_1(t)^2 dt \quad (5)$$

This is an optimal control problem and can be solved using Pontryagin's maximum (also called minimum) principle [6]. First form the Hamiltonian:

[1]Efficiency of charging an ultracapacitor from initial state of charge SOC_i to final state of charge SOC_f with a constant voltage source is:

$$\rho = \frac{1}{1 + \frac{(1-SOC_i)^2}{SOC_f^2 - SOC_i^2}}$$

$$H(x_1, u_1, t) = Ru_1(t)^2 + \lambda_1 \frac{u_1}{CE_{max}} \quad (6)$$

where λ_1 is a co-state. The optimal co-state should satisfy the following dynamic equation:

$$\frac{d}{dt}\lambda_1 = -\frac{\partial H}{\partial x_1} = 0 \quad (7)$$

implying that in this problem the optimal λ_1 must be a constant. The unconstrained optimal solution will also satisfy the following condition:

$$\frac{\partial H}{\partial u_1} = 0 \rightarrow u_1(t) = -\frac{1}{2}\frac{1}{RCE_{max}}\lambda_1 \quad (8)$$

showing that the optimal input (charging current) must be a constant. One can now integrate Eq. (4) and use the boundary conditions $x_1(0) = SOC_i$ and $x_1(t_f) = SOC_f$ to find the value of this optimal and constant input:

$$u_1^*(t) = \frac{C.E_{max}.(SOC_f - SOC_i)}{t_f} \quad (9)$$

where * denotes the optimal solution. This is in fact the minimizing solution since $\frac{\partial^2 H}{\partial u^2} > 0$. The optimal charging efficiency, ρ^*, is

$$\rho^* = \frac{\frac{1}{2}CE_{max}^2(SOC_f^2 - SOC_i^2)}{\int_0^{t_f} Ru_1^*(t)^2 dt + \frac{1}{2}CE_{max}^2(SOC_f^2 - SOC_i^2)},$$

Substituting for u_1^* from (9) yields:

$$\rho^* = \frac{1}{1 + (\frac{2RC}{t_f}).(\frac{SOC_f - SOC_i}{SOC_f + SOC_i})} \quad (10)$$

which implies larger charging times and/or smaller RC values improve the charging efficiency. When t_f approaches infinity the charging efficiency approaches 1, which is a 100% improvement over the case with constant voltage charging. Another interesting observation is that charging from a higher initial state of charge will result in higher efficiencies. Note that we assumed that both the input u_1 and the state x_1 were unconstrained. Treatment of the constrained case in this problem is straightforward.

Still, one cannot assume that constant current charging is the energy optimal approach during a regenerative braking event: i) In braking from a given speed to a desired final speed, the available charging energy is fixed; different from the implicit assumption in the above derivations that an unlimited source of charging energy exists, ii) Losses are not only resistive but also due to friction brakes if applied, iii) When braking from high speeds, available kinetic energy may be much more than the capacity of the ultracapacitor; therefore minimizing resistive and friction braking losses may not be even necessary. And iv) The optimal charging profile is influenced by the electric motor characteristics.

Next we formulate a new optimal control problem to address the particular nature of regeneration. To understand the subsystem level efficiency, we first ignore the motor dynamics and losses and focus only on the subsystem shown in Figure (1).

III. Formulation of Optimal Charging During Regeneration - Excluding Motor Dynamics and Constraints

Going back to Figure (1) the following relationship is observed:

$$E_{dc}(t) = E_{max}.SOC(t) + R.i_{chg} = E_{max}.x_1 + R.u_1 \quad (11)$$

where E_{dc} is the voltage of the power electronic device on the ultracapacitor side. This is related to the available regeneration power P_{regen} as follows:

$$P_{regen}(t) = \frac{1}{\eta(x_1,u_1)} E_{dc}(t).i_{chg} = \frac{(E_{max}.x_1 + R.u_1).u_1}{\eta(x_1,u_1)} \quad (12)$$

where $0 < \eta \leqslant 1$ is the charging efficiency of the dc/dc converter and in general a function of its current and voltage, therefore $\eta(x_1,u_1)$.

In addition to the state of charge which is a dynamic state, the vehicle velocity also constitutes a state. The governing equation for velocity $x_2 = v$ can be written using Newton's second law of motion:

$$\frac{d}{dt}x_2 = -\frac{1}{m} \cdot \frac{P_{regen} + P_{friction}}{x_2} \quad (13)$$

$$\frac{d}{dt}x_2 = -\frac{1}{mx_2}\left(\frac{E_{max}.x_1 + R.u_1}{\eta(x_1,u_1)}u_1 + u_2\right) \quad (14)$$

where $u_2 = P_{friction}$ is the power dissipated by friction braking and considered a second control input and m is the total mass of the vehicle. For a regeneration period we need to require that:

$$u_1(t) \geqslant 0, \quad u_2(t) \geqslant 0 \quad (15)$$

Given a braking event during which the vehicle speed is reduced from $x_2(0) = v_i$ to $x_2(t_f) = v_f$ the goal is to maximize the regeneration efficiency. The objective function to be *maximized* then can be chosen to be the charge stored in the ultracapacitor at the end of the braking event at time t_f:

$$J = x_1(t_f) \quad (16)$$

The objective is to find the control inputs $u_1(t)$ and $u_2(t)$ that *maximize* J subject to the input constraints in (15). The equality constraints to be satisfied are the state dynamics in Equations (4) and (14). The boundary conditions for the case when the ultracapacitor is initially discharged are:

$$\begin{aligned} x_1(0) &= 0, \quad x_1(t_f) \leqslant 1 \\ x_2(0) &= v_i, \quad x_2(t_f) = v_f \end{aligned} \quad (17)$$

Because of the monotonically increasing state of charge in the ultracapacitor during regeneration, the imposed terminal constraint of $x_1(t_f) \leqslant 1$ satisfies also the requirement that $x_1(t) \leqslant 1 \ \forall \ t \in [0, \ t_f]$. The braking time t_f can be assumed to be fixed or left to be free with an upper bound, i.e. $t_f \leqslant t_{max}$.

Practical limitations may impose constraints on maximum allowable current, dc/dc converter output voltage, and maximum regeneration and braking powers or forces. Such hard constraints must be included a-priori when solving the optimal control problem analytically or numerically.

Following a standard optimal control routine, one first forms the Hamiltonian:

$$H = \lambda_1 \frac{u_1}{CE_{max}} - \lambda_2 \frac{1}{mx_2}\left(\frac{E_{max}.x_1 + R.u_1}{\eta(x_1,u_1)}u_1 + u_2\right) \quad (18)$$

where λ_1 and λ_2 are the co-states. Because in the operational range of a dc/dc converter, its efficiency does not vary significantly, here we assume the dc/dc converter efficiency is constant, i.e. $\eta(x_1,u_1) = \eta$. This assumption reduces the complexity in the following derivations. Subsequently the differential equations governing the dynamics of the co-states λ_1 and λ_2 can be found to be:

$$\frac{d}{dt}\lambda_1 = -\frac{\partial H}{\partial x_1} = \frac{E_{max}}{m\eta}\frac{\lambda_2}{x_2}u_1 \quad (19)$$

$$\frac{d}{dt}\lambda_2 = -\frac{\partial H}{\partial x_2} = \frac{1}{m}\frac{\lambda_2}{x_2{}^2}\left(\frac{E_{max}.x_1 + R.u_1}{\eta}u_1 + u_2\right) \quad (20)$$

The optimal solution should satisfy the co-state equations (19) and (20) in addition to the state equations (4) and (14). The boundary conditions are the three equalities in (17) and also after observing $x_1(t_f)$ is free, on can obtain $\lambda_1(t_f) = 1$ (more details can be found in [6]). In the absence of any input or state constraints, the optimal solution(s) should also satisfy $\frac{\partial H}{\partial u_1} = 0$ and $\frac{\partial H}{\partial u_2} = 0$ from which one obtains a relationship between the control inputs and the states and the co-states. In the above problem the control inputs are constrained as shown in (15), so we must resort to Pontryagin's maximum principle instead:

$$H(\mathbf{x}^*(t), \lambda^*(t), \mathbf{u}^*(t), t) \geqslant H(\mathbf{x}^*(t), \lambda^*(t), \mathbf{u}(t), t), \ \forall \, t \in [0, \ t_f] \quad (21)$$

where * denotes the optimal solution, and the vectors $\mathbf{x}$, λ, $\mathbf{u}$ stack the states, the co-states, and the control inputs respectively. In other words the optimal control input should maximize the Hamiltonian given the input constraints. As opposed to the case of direct charging shown in Section II, the necessary conditions for optimality are more complex and it is not clear what the optimal charging profile is.

This is a two-point boundary value problem which is generally hard to solve. In the simulations of this paper, we employ the software package PROPT [7] that uses pseudospectral collocation methods to obtain a candidate optimal solution. The solution is mathematically equivalent to one obtained from application of Pontryagin's Maximum Principle; and satisfies the necessary conditions for optimality, but not guaranteed to be the globally optimal solution.

A. Simulation Results

The mass of the vehicle was assumed to be 9700 kg corresponding to the heavy hybrid electric truck simulated in [2]. The energy storage is a 145 Volt ultracapacitor bank with total capacitance of $C = 56$ Farads and lumped line resistance of $R = 0.07$ Ohms[2]. These values are adopted from previous work in our group where performance of an ultracapacitor-assisted heavy truck had been studied over a whole drive cycle [2]. To isolate the influence of the DC/DC converter, its efficiency η is assumed to be 1.

Figure 2 shows the (candidate) optimal solution, obtained using the software package PROPT [7], for a first case study. The ultracapacitor had zero initial charge and the car was stopped from an initial velocity in a maximum of 20 seconds ($0 \leqslant t_f \leqslant 20$). The only constrained inputs are those in (15) on the control inputs. The initial velocity is chosen such that the available kinetic energy is equal to the energy storage capacity of the ultracapacitor, i.e.

$$\frac{1}{2}Mv_i^2 = \frac{1}{2}CE_{max}^2(SOC_f^2 - SOC_i^2)$$

$$\rightarrow v_i = E_{max}\sqrt{\frac{C.(SOC_f^2 - SOC_i^2)}{M}} = 11\ m/s$$

Interestingly, the optimal solution found seems to be a constant current solution $i_{chg}(t) \approx 344$ Amperes; with the optimal final time equaling its maximum of 20 seconds. This charges up the empty ultracapacitor to the final state of the charge of $SOC_f = 0.8475$ yielding a regeneration efficiency of 72%. These numbers are consistent with those in equations (9) and (10). A lower line resistance or increasing the charge time will results in increased charging efficiency according to the relationship in (10). Here the friction braking power is practically equal to zero as expected. We note that if the current that the system can accept is lower than 344 Amperes, current constraints should be imposed, in which case friction brakes will be activated and the regeneration efficiency will be lower than 72%. A constrained current simulation case study (not shown here) revealed that, if only current was constrained, the optimal charing current would remain constant but at the constraint boundary. Deceleration limits and upper bounds on regenerative and friction braking powers may also be imposed if needed. Limiting the deceleration rate to $-1m/s^2$ did not have a considerable influence on the results and on the efficiency reported above; it only limited the charging current and power at a fraction of the last second of the simulation when the velocity was nearly zero.

Another interesting observation is that the resulting optimal velocity profile is not a linear function of time. This is different from linear slow-down profiles in many standard drive cycles. In the above simulation, imposing a linearly decreasing profile, $v(t) = v_i(1 - \frac{t}{t_f})$, will require very high charging currents initially ($i_{chg} > 500$ Amps for the first 5 seconds). Even if such large currents can be allowed, the regeneration efficiency will only be 62% which is 10% lower than the optimal efficiency. If the current has an upper bound, friction brakes will need to be activated to maintain a linearly decreasing velocity, reducing the regeneration efficiency to below 62%.

[2]Note that this value is meant to include the resistance of the motor and connecting wires and may be an overestimate. The internal resistance of the ultracapacitor module is much lower.

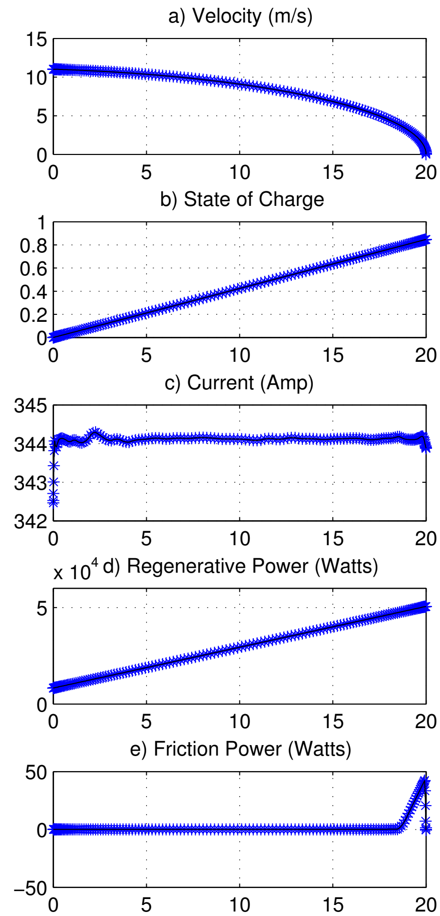

Fig. 2. Case Study I: The ultracapacitor is initially empty of charge.

The above simulation was repeated with initial ultracapacitor state of charge of 0.5. The initial velocity in this case is chosen at nearly 9 m/s such that the available kinetic energy is equal to the remaining energy storage capacity of the ultracapacitor. Figure 3 shows the simulation results. The charging current profile is constant as expected and the charging efficiency in this case is much higher and nearly 90%, indicating operating an ultracapacitor in a higher and narrower state of charge range has efficiency advantages.

IV. Formulation of Optimal Charging During Regeneration - Including Motor Dynamics and Constraints

The above results are based on the assumption that the regeneration power P_{regen} in Equation (13) does not have speed dependent bounds. Consequently one observes large values of regeneration power at low speeds in Figures 2

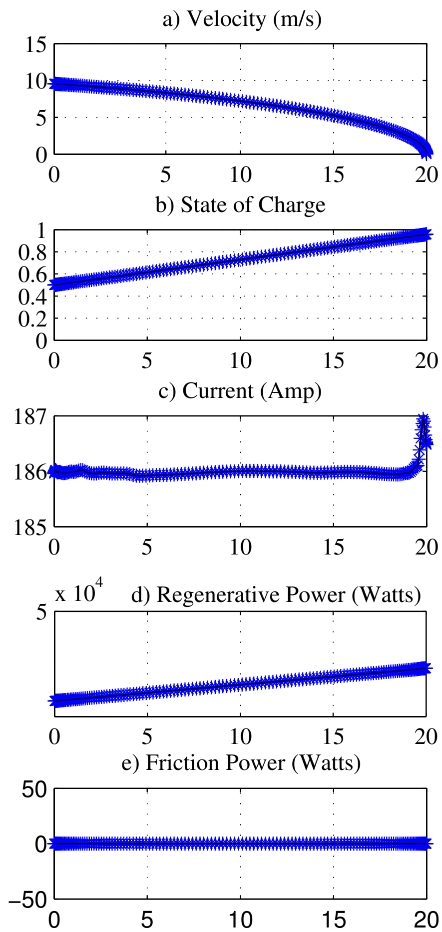

Fig. 3. Case Study II: The ultracapacitor is initially 50% charged.

and 3. This assumption is not realistic as the regeneration capability of an electric motor significantly reduces at low speeds. The power regenerated by a DC motor can be calculated by:

$$P_{regen} = (K_t\omega_m - R_m i_m - L_m \frac{di_m}{dt})i_m \quad (22)$$

where i_m, R_m, L_m, and K_t are the motor current, resistance, inductance, and back-emf constant respectively. The rotational speed of the motor ω_m is proportional to vehicle speed, $\omega_m = \frac{v}{r_w}$, where r_w is the driven wheel radius. Obviously the maximum regeneration power is reduced as the vehicle speed drops.

To account for the constraints and losses due to the motor, the motor dynamics should be augmented to the equations developed in the previous section by introducing a third state variable $x_3 = i_m$. Substituting for P_{regen} from Eq. (12) in Eq. (22) and rearranging yield:

$$\frac{d}{dt}x_3 = -\frac{1}{\eta L_m}(E_{max}.x_1 + R.u_1)\frac{u_1}{x_3} - \frac{R_m}{L_m}x_3 + \frac{K_t}{L_m}\frac{x_2}{r_w} \quad (23)$$

where the regeneration mode is enforced by requiring $x_3 > 0$. The optimal control problem formulations remains the same as that in Section III; only the motor state dynamics and the state constraint are added. We again use PROPT to obtain a solution that satisfies the necessary conditions for optimality.

A. *Preliminary Simulation Results*

A preliminary simulation case study was performed to illustrate the impact of the motor on optimal regeneration efficiency and on optimal charging profile. A DC motor with $R_m = 0.02$ Ohm, $L_m = 0.0025$ Henry, and $K_t = 30$ Volts per radians/sec was used. It was assuming that this motor is directly connected to the wheels of radius $r_w = 0.57$ m matching the truck in [2]. Other parameters of the energy storage system are those reported in Section III-A. These parameter values are not necessarily optimal or even realistic and are for preliminary illustration of the impact of the motor. Future work is needed for appropriately sizing and selecting a DC or AC motor that best matches the energy storage system.

The ultracapacitor's initial state of charge is assumed to be 0.5 and the initial velocity is chosen to be nearly 9 m/s. Our preliminary results in Figure 4 indicate that the charging current profile is no more a constant profile, the regeneration power drops at low speeds and friction brakes are activated. The regeneration efficiency is now 87%, lower than the result of case II in Section III-A. The optimal velocity profile is closer to a line but not completely linear.

V. Summary and Future Work

Previous research has studied the positive impact that ultracapacitor-assist can have on fuel economy of vehicles over entire drive cycles. The goal of this paper was to provide more insight into the efficiency bottlenecks of an ultracapacitor energy storage system by focusing on an isolated regenerative braking event and using a bottom-up subsystem-level approach. The objective of maximizing energy recuperation was formulated in an optimal control framework and candidate optimal charging profiles were calculated based on Pontryagin's Maximum Principle. We showed, analytically, that an ultracapacitor can be charged more efficiently and with lower currents if operated in a narrower and higher state of charge band. This implies that a larger ultracapacitor operated at higher state of charge can be charged more efficiently than a smaller size ultracapacitor. Because larger capacitance or resistance (RC) negatively influence the rapid charging efficiency; the trade-off between the ultracapacitor size and its operating range should be observed and will be addressed in our future work. The motor and the dc/dc converter specifications play an important role in the overall efficiency and future work can benefit from the approach in this paper for appropriately sizing each component. We also observed that linearly decreasing velocity profiles, as commonly seen in standard drive cycles, do not necessarily lead to the best regeneration efficiency.

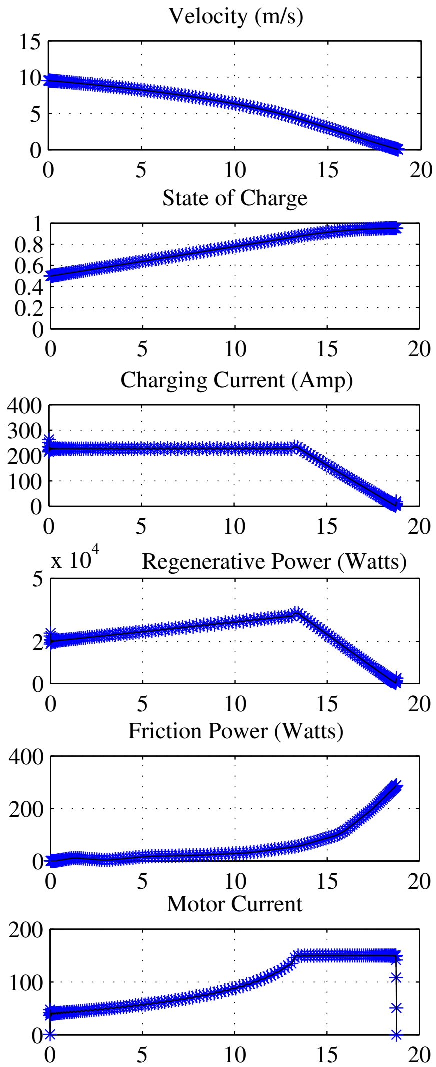

Fig. 4. Case Study III: Motor dynamics included. The ultracapacitor is initially 50% charged.

VI. Acknowledgements

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